Discovering Geometry

Correlation to the Common Core State Standards for Mathematics

Standards for Mathematical Practice	
Standard	***Discovering Geometry* Lessons**
1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.	The Standards for Mathematical Practice are addressed in both the daily Investigations and the Exercise Sets in every lesson of *Discovering Geometry*. Because rich, non-routine problems are integral to the curriculum, students gain proficiency and comfort with analyzing problems, trying different solution methods, and evaluating their answers. These standards are discussed in more depth on pages xxviii to xxxi.

Number and Quantity	
Standard	***Discovering Geometry* Lessons**
The Real Number System	
Extend the properties of exponents to rational exponents	
N.RN.1 Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.	
N.RN.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents.	
Use properties of rational and irrational numbers	
N.RN.3 Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.	
Quantities*	
Reason quantitatively and use units to solve problems	
N.Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.	
N.Q.2 Define appropriate quantities for the purpose of descriptive modeling.	
N.Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.	
The Complex Number System	
Perform arithmetic operations with complex numbers	
N.CN.1 Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.	
N.CN.2 Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.	
N.CN.3 (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.	

Number and Quantity	
Standard	***Discovering Geometry* Lessons**
Represent complex numbers and their operations on the complex plane	
N.CN.4 (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.	
N.CN.5 (+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation.	
N.CN.6 (+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.	
Use complex numbers in polynomial identities and equations	
N.CN.7 Solve quadratic equations with real coefficients that have complex solutions.	
N.CN.8 (+) Extend polynomial identities to the complex numbers.	
N.CN.9 (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.	
Vector and Matrix Quantities	
Represent and model with vector quantities	
N.VM.1 (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., $\boldsymbol{v}$, $\lvert\boldsymbol{v}\rvert$, $\lVert\boldsymbol{v}\rVert$, v).	Lesson 5.5: Properties of Parallelograms
N.VM.2 (+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.	Lesson 7.2: Properties of Isometries Lesson 7.3: Compositions of Transformations Lesson 12.5: Problem Solving with Trigonometry
N.VM.3 (+) Solve problems involving velocity and other quantities that can be represented by vectors.	Lesson 12.5: Problem Solving with Trigonometry
Perform operations on vectors	
N.VM.4 (+) Add and subtract vectors.	
N.VM.4a Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.	Lesson 5.5: Properties of Parallelograms Lesson 12.5: Problem Solving with Trigonometry
N.VM.4b Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.	Lesson 12.5: Problem Solving with Trigonometry
N.VM.4c Understand vector subtraction $\boldsymbol{v} - \boldsymbol{w}$ as $\boldsymbol{v} + (-\boldsymbol{w})$, where $-\boldsymbol{w}$ is the additive inverse of $\boldsymbol{w}$, with the same magnitude as $\boldsymbol{w}$ and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.	
N.VM.5 (+) Multiply a vector by a scalar.	
N.VM.5a (+) Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$.	
N.VM.5b (+) Compute the magnitude of a scalar multiple $c\boldsymbol{v}$ using $\lVert c\boldsymbol{v}\rVert = \lvert c\rvert\boldsymbol{v}$. Compute the direction of $c\boldsymbol{v}$ knowing that when $\lvert c\rvert\boldsymbol{v} \neq 0$, the direction of $c\boldsymbol{v}$ is either along $\boldsymbol{v}$ (for $c > 0$) or against $\boldsymbol{v}$ (for $c < 0$).	

Number and Quantity

Standard	*Discovering Geometry* Lessons
Perform operations on matrices and use matrices in applications	
N.VM.6 (+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.	
N.VM.7 (+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.	
N.VM.8 (+) Add, subtract, and multiply matrices of appropriate dimensions.	Lesson 7.3: Compositions of Transformations (Exercise 18) Lesson 7.7: Tessellations That Use Rotation (Exercise 14) Chapter 7 Review (Take Another Look Exercises 4 and 5)
N.VM.9 (+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.	
N.VM.10 (+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.	
N.VM.11 (+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.	
N.VM.12 (+) Work with 2×2 matrices as a transformations of the plane, and interpret the absolute value of the determinant in terms of area.	Chapter 7 Review (Take Another Look Exercises 4 and 5)

Algebra

Standard	*Discovering Geometry* Lessons
Seeing Structure in Expressions	
Interpret the structure of expressions	
A.SSE.1 Interpret expressions that represent a quantity in terms of its context.*	
A.SSE.1a Interpret parts of an expression, such as terms, factors, and coefficients.	Using Your Algebra Skills 12: Transforming Functions
A.SSE.1b Interpret complicated expressions by viewing or more of their parts as a single entity.	Using Your Algebra Skills 12: Transforming Functions
A.SSE.2 Use the structure of an expression to identify ways to rewrite it.	
Write expressions in equivalent forms to solve problems	
A.SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.	
A.SSE.3a Factor a quadratic expression to reveal the zeros of the function it defines.	
A.SSE.3b Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.	
A.SSE.3c Use the properties of exponents to transform expressions for exponential functions.	
A.SSE.4 Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.	
Arithmetic with Polynomials and Rational Expressions	
Perform arithmetic operations on polynomials	
A.APR.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.	

Algebra	
Standard	***Discovering Geometry* Lessons**
Understand the relationship between zeros and factors of polynomials	
A.APR.2 Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.	
A.APR.3 Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.	
Use polynomial identities to solve problems	
A.APR.4 Prove polynomial identities and use them to describe numerical relationships.	
A.APR.5 (+) Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.	
Rewrite rational expressions	
A.APR.6 Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.	
A.APR.7 (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.	
Creating Equations*	
Create equations that describe numbers or relationships	
A.CED.1 Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*	Using Your Algebra Skills 5: Writing Equations
A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.	Using Your Algebra Skills 6: Solving Systems of Linear Equations Using Your Algebra Skills 7: Finding Points of Concurrency
A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.	Using Your Algebra Skills 6: Solving Systems of Linear Equations Using Your Algebra Skills 7: Finding Points of Concurrency
A.CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.	Using Your Algebra Skills 10: Solving for Any Variable
Reasoning with Equations and Inequalities	
Understand solving equations as a process of reasoning and explain the reasoning	
A.REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.	Using Your Algebra Skills 4: Solving Equations Using Your Algebra Skills 6: Solving Systems of Linear Equations Using Your Algebra Skills 7: Finding Points of Concurrency
A.REI.2 Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.	
Solve equations and inequalities in one variable	
A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.	Using Your Algebra Skills 4: Solving Equations Using Your Algebra Skills 10: Solving for Any Variable
A.REI.4 Solve quadratic equations in one variable.	
A.REI.4a Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.	

Algebra

Standard	*Discovering Geometry* Lessons
A.REI.4b Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.	(Partial) Using Your Algebra Skills 8: Products, Factors, and Quadratic Equations
Solve systems of equations	
A.REI.5 Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.	Using Your Algebra Skills 6: Solving Systems of Linear Equations
A.REI.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.	Using Your Algebra Skills 6: Solving Systems of Linear Equations Using Your Algebra Skills 7: Finding Points of Concurrency
A.REI.7 Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.	
A.REI.8 (+) Represent a system of linear equations as a single matrix equation in a vector variable.	
A.REI.9 (+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3×3 or greater).	
Represent and solve equations and inequalities graphically	
A.REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).	Using Your Algebra Skills 12: Transforming Functions
A.REI.11 Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.*	
A.REI.12 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.	

Functions

Standard	*Discovering Geometry* Lessons
Interpreting Functions	
Understand the concept of a function and use function notation	
F.IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.	
F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.	Lesson 2.2: Finding the nth Term Using Your Algebra Skills 12: Transforming Functions
F.IF.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.	Lesson 2.2: Finding the nth Term
Interpret functions that arise in applications in terms of the context	
F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.	

Functions	
Standard	***Discovering Geometry* Lessons**
F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.	
F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.*	
Analyze functions using different representations	
F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.*	
F.IF.7a Graph linear and quadratic functions and show intercepts, maxima, and minima.	
F.IF.7b Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.	
F.IF.7c Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.	
F.IF.7d (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.	
F.IF.7e Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.	
F.IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.	
F.IF.8a Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.	
F.IF.8b Use the properties of exponents to interpret expressions for exponential functions.	
F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).	
Building Functions	
Build a function that models a relationship between two quantities	
F.BF.1 Write a function that describes a relationship between two quantities.*	
F.BF.1a Determine an explicit expression, a recursive process, or steps for calculation from a context.	Lesson 2.2: Finding the *n*th Term
F.BF.1b Combine standard function types using arithmetic operations.	
F.BF.1c (+) Compose functions.	
F.BF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.*	
Build new functions from existing functions	
F.BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.	Using Your Algebra Skills 12: Transforming Functions
F.BF.4 Find inverse functions.	
F.BF.4a Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse.	
F.BF.4b (+) Verify by composition that one function is the inverse of another.	
F.BF.4c (+) Read values of an inverse function from a graph or a table, given that the function has an inverse.	

Functions	
Standard	***Discovering Geometry* Lessons**
F.BF.4d (+) Produce an invertible function from a non-invertible function by restricting the domain.	
F.BF.5 (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.	
Linear and Exponential Models	
Construct and compare linear and exponential models and solve problems	
F.LE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions.	
F.LE.1a Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.	
F.LE.1b Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.	
F.LE.1c Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.	
F.LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).	Using Your Algebra Skills 5: Writing Linear Equations
F.LE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.	
F.LE.4 For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.	
Interpret expressions for functions in terms of the situation they model	
F.LE.5 Interpret the parameters in a linear or exponential function in terms of a context.	
Trigonometric Functions	
Extend the domain of trigonometric functions using the unit circle	
F.TF.1 Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.	
F.TF.2 Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.	
F.TF.3 (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi - x$, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number.	
F.TF.4 (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.	
Model periodic phenomena with trigonometric functions	
F.TF.5 Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.*	
F.TF.6 (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.	
F.TF.7 (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.	

Functions

Standard	*Discovering Geometry* Lessons
Prove and apply trigonometric identities	
F.TF.8 Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle.	
F.TF.9 (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.	

Modeling

Modeling Standards

Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (*).

Geometry

Standard	*Discovering Geometry* Lessons
Congruence	
Experiment with transformations in the plane	
G.CO.1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.	Lesson 1.1: Building Blocks of Geometry Lesson 1.2: Poolroom Math Lesson 1.3: What's a Widget Lesson 1.4: Polygons Lesson 1.7: Circles
G.CO.2 Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).	Lesson 1.7: Circles (Exercises 19–21) Lesson 2.3: Mathematical Modeling Lesson 2.4: Deductive Reasoning Chapter 2 Exploration: Patterns in Fractals Lesson 4.2: Properties of Isosceles Triangles (Exercises 24, 25) Lesson 7.1: Transformations and Symmetry Lesson 7.2: Properties of Isometries Lesson 7.3: Compositions of Transformations Lesson 11.1: Similar Polygons Lesson 11.2: Similar Triangles (Exercises 19, 20) Chapter 11 Exploration: Constructing a Dilation Design
G.CO.3 Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.	Symmetry is first introduced in Lesson 0.1, and then reviewed periodically as part of developing students' visualization skills before deeper coverage in Lesson 7.1. Lesson 0.1: Geometry in Nature and in Art Lesson 1.4: Polygons Lesson 1.6: Special Quadrilaterals (Exercise 16) Lesson 1.7: Circles Chapter 2 Exploration: Patterns in Fractals Lesson 2.1: Inductive Reasoning (Exercise 42) Lesson 2.5: Angle Relationships (Exercise 14) Lesson 3.1: Duplicating Segments and Angles (Exercise 14) Lesson 5.3: Kite and Trapezoid Properties Lesson 7.1: Transformation and Symmetry Lesson 7.7: Tessellations That Use Rotations

Geometry	
Standard	***Discovering Geometry* Lessons**
G.CO.4 Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines and line segments.	Lesson 7.1: Transformations and Symmetry Lesson 7.2: Properties of Isometries Lesson 7.3: Compositions of Transformations Lesson 7.6: Tessellations Using Only Translations Lesson 7.7: Tessellations That Use Rotations
G.CO.5 Given a geometric figure and a rotation, reflection or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.	Lesson 7.1: Transformations and Symmetry Lesson 7.2: Properties of Isometries Lesson 7.3: Compositions of Transformations Lesson 11.1: Similar Polygons Lesson 11.2: Similar Triangles (Exercises 19, 20) Chapter 11 Exploration: Constructing a Dilation Design
Understand congruence in terms of rigid motions	
G.CO.6 Use geometric descriptions of rigid motions to transform figures and to predict the effect of a rigid motion on a figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.	Patty paper and compass-and-straightedge constructions prepare students for the formal introduction to rigid motions in Lesson 7.1. For example, see: Lesson 2.6: Special Angles on Parallel Lines Lesson 3.1: Duplicating Segments and Angles Lesson 3.6: Construction Problems Lesson 7.1: Transformations and Symmetry
G.CO.7 Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.	Lesson 3.6: Construction Problems Lesson 4.4: Are There Congruence Shortcuts? Lesson 4.5: Are There Other Congruence Shortcuts?
G.CO.8 Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.	Lesson 3.6: Construction Problems Lesson 4.4: Are There Congruence Shortcuts? Lesson 4.5: Are There Other Congruence Shortcuts?
Prove geometric theorems	
G.CO.9 Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.*	Lesson 2.5: Angle Relationships Lesson 2.6: Special Angles on Parallel Lines Lesson 3.2: Constructing Perpendicular Bisectors Lesson 5.1: Polygon Sum Conjecture Lesson 5.2: Exterior Angles of a Polygon Lesson 5.3: Kite and Trapezoid Properties Lesson 13.2: Planning a Geometry Proof
G.CO.10 Prove theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.*	Lesson 3.8: The Centroid Chapter 3 Exploration: The Euler Line Lesson 4.1: Triangle Sum Conjecture Lesson 4.2: Properties of Isosceles Triangles Lesson 4.3: Triangle Inequalities Lesson 5.4: Properties of Midsegments Lesson 13.3: Triangle Proofs
G.CO.11 Prove theorems about parallelograms. *Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other and conversely, rectangle are parallelograms with congruent diagonals.*	Lesson 5.5: Properties of Parallelograms Lesson 5.6: Properties of Special Parallelograms Lesson 5.7: Proving Quadrilateral Properties Lesson 13.4: Quadrilateral Proofs Using Your Algebra Skills 13: Coordinate Proof

Geometry	
Standard	***Discovering Geometry* Lessons**
Make geometric constructions	
G.CO.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.*	Throughout *Discovering Geometry* students construct with compass and straightedge and patty paper folding. Dynamic geometry constructions are incorporated both as Explorations in the student book and as demonstrations and replacement lessons in the ancillary *Discovering Geometry with The Geometer's Sketchpad*. See, for example: Lesson 3.1: Duplicating Segments and Angles Lesson 3.2: Constructing Perpendicular Bisectors Lesson 3.3: Constructing Perpendiculars to a Line Lesson 3.4: Constructing Angle Bisectors Lesson 3.6: Construction Problems Chapter 6 Exploration: Intersecting Lines Through a Circle Chapter 13 Exploration: Proof as Challenge and Discovery
G.CO.13 Construct an equilateral triangle, a square and a regular hexagon inscribed in a circle.	Lesson 0.3: Circle Designs Lesson 1.7: Circles (Exercise 17) Lesson 3.1: Duplicating Segments and Angles (Exercises 8 and 9) Lesson 3.3: Constructing Perpendiculars to a Line (Exercise 10) Lesson 3.5: Constructing Parallel Lines (Exercise 3)
Similarity, Right Triangles, and Trigonometry	
Understand similarity in terms of similarity transformations	
G.SRT.1 Verify experimentally the properties of dilations given by a center and a scale factor:	
G.SRT.1a A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.	(Partial) Lesson 11.1: Similar Polygons
G.SRT.1b The dilation of a line segment is longer or shorter in the ratio given by the scale factor.	Lesson 11.1: Similar Polygons Lesson 11.2: Similar Triangles (Exercises 19, 20) Chapter 11 Exploration: Constructing a Dilation Design
G.SRT.2 Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all pairs of angles and the proportionality of all pairs of sides.	Lesson 11.1: Similar Polygons Lesson 11.2: Similar Triangles
G.SRT.3 Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.	Lesson 11.2: Similar Triangles
Prove theorems involving similarity	
G.SRT.4 Prove theorems about triangles. *Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean theorem proved using triangle similarity.*	Lesson 11.3: Indirect Measurement with Similar Triangles Lesson 11.4: Corresponding Parts of Similar Triangles Lesson 11.7: Proportional Segments Between Parallel Lines Chapter 12 Exploration: Indirect Measurement Lesson 13.7: Similarity Proofs
G.SRT.5 Use triangle congruence and similarity criteria to solve problems and to prove relationships in geometric figures.	Lesson 4.4: Are There Congruence Shortcuts? Lesson 4.5: Are There Other Congruence Shortcuts? Lesson 4.6: Corresponding Parts of Congruent Triangles Lesson 4.7: Flowchart Thinking Lesson 4.8: Proving Special Triangle Conjectures Chapter 11: Similarity Chapter 12 Exploration: Indirect Measurement Lesson 13.3: Triangle Proofs Lesson 13.7: Similarity Proofs

Geometry	
Standard	***Discovering Geometry* Lessons**
Define trigonometric ratios and solve problems involving right triangles	
G.SRT.6 Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.	Lesson 12.1: Trigonometric Ratios
G.SRT.7 Explain and use the relationship between the sine and cosine of complementary angles.	Lesson 12.1: Trigonometric Ratios
G.SRT.8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.	Lesson 12.1: Trigonometric Ratios Lesson 12.2: Problem Solving with Right Triangles
(+) Apply trigonometry to general triangles	
G.SRT.9 (+) Derive the formula $A = \frac{1}{2}ab\sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side	Lesson 12.3: The Law of Sines
G.SRT.10 (+) Prove the Laws of Sines and Cosines and use them to solve problems.	Lesson 12.3: The Law of Sines Lesson 12.4: The Law of Cosines Chapter 12 Review: Take Another Look 5
G.SRT.11 (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).	Lesson 12.3: The Law of Sines Lesson 12.4: The Law of Cosines Lesson 12.5: Problem Solving with Trigonometry
Circles	
Understand and apply theorems about circles	
G.C.1 Prove that all circles are similar.	*Teacher's Edition*, p. 609
G.C.2 Identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*	Lesson 6.1: Tangent Properties Lesson 6.2: Chord Properties Lesson 6.3: Arcs and Angles
G.C.3 Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.	Lesson 3.7: Constructing Points of Concurrency Chapter 3 Exploration: The Euler Line Lesson 6.3: Arcs and Angles Lesson 13.6: Circle Proofs
G.C.4 (+) Construct a tangent line from a point outside a given circle to the circle.	Lesson 6.1: Tangent Properties
Find arc lengths and areas of sectors of circles	
G.C.5 Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.	Lesson 6.7: Arc Length Lesson 8.5: Areas of Circles Lesson 8.6: Any Way You Slice It
Expressing Geometric Properties with Equations	
Translate between the geometric description and the equation for a conic section	
G.GPE.1 Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.	Lesson 9.5: Distance in Coordinate Geometry
G.GPE.2 Derive the equation of a parabola given a focus and directrix.	
G.GPE. 3 (+) Derive the equations of ellipses and hyperbolas given the foci using the fact that the sum or difference of distances from the foci is constant.	
Use coordinates to prove simple geometric theorems algebraically	
G.GPE.4 Use coordinates to prove simple geometric theorems algebraically.	Using Your Algebra Skills 13: Coordinate Proof

Geometry	
Standard	***Discovering Geometry* Lessons**
G.GPE.5 Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).	Using Your Algebra Skills 3: Slopes of Parallel and Perpendicular Lines Using Your Algebra Skills 13: Coordinate Proof
G.GPE.6 Find the point on a directed line segment between two given points that partitions the segment in a given ratio.	Lesson 11.6: Proportions with Volume (Exercise 20) Lesson 11.7: Proportional Segments Between Parallel Lines
G.GPE.7 Use coordinates to compute perimeters of polygons and areas for triangles and rectangles, e.g. using the distance formula.*	Lesson 9.5: Distance in Coordinate Geometry
Geometric Measurement and Dimension	
Explain volume formulas and use them to solve problems	
G.GMD.1 Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments.*	Lesson 6.4: Proving Circle Conjectures Lesson 6.5: The Circumference/Diameter Ratio Lesson 6.6: Around the World Lesson 8.5: Areas of Circles Lesson 10.2: Volume of Prisms and Cylinders Lesson 10.3: Volume of Pyramids and Cones
G.GMD.2 (+) Given an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.	Cavalieri's principle is discussed in the *Discovering Geometry Teacher's Edition* (pages 532, 539, and 559). Lesson 10.2: Volume of Prisms and Cylinders Lesson 10.3: Volume of Pyramids and Cones Lesson 10.6: Volume of a Sphere
G.GMD.3 Use volume formulas for cylinders, pyramids, cones and spheres to solve problems.*	Lesson 10.2: Volume of Prisms and Cylinders Lesson 10.3: Volume of Pyramids and Cones Lesson 10.4: Volume Problems Lesson 10.5: Displacement and Density Lesson 10.6: Volume of a Sphere
Visualize relationships between two-dimensional and three-dimensional objects	
G.GMD.4 Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.	Visualization skills are emphasized throughout the book. Examples include: Lesson 1.8: Space Geometry Lesson 1.9: A Picture is Worth a Thousand Words Chapter 1 Review Lesson 2.1: Inductive Reasoning (Exercises 23, 24) Lesson 10.2: Volume of Prisms and Cylinders (Exercises 22, 23)
Modeling with Geometry	
Apply geometric concepts in modeling situations	
G.MG.1 Use geometric shapes, their measures and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).*	Lesson 10.1: The Geometry of Solids Lesson 10.2: Volume of Prisms and Cylinders Lesson 10.3: Volume of Pyramids and Cones Lesson 10.4: Volume Problems Lesson 10.5: Displacement and Density Lesson 10.6: Volume of a Sphere Lesson 10.7: Surface Area of a Sphere
G.MG.2 Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).*	Lesson 10.5: Displacement and Density

Geometry

Standard	*Discovering Geometry* Lessons
G.MG.3 Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy constraints or minimize cost; working with typographic grid systems based on ratios).*	Chapter 5 Project: Building an Arch, page 280 Chapter 5 Project: Japanese Puzzle Quilts, page 303 Chapter 6 Project: Racetrack Geometry, page 354 Chapter 7 Project: Kaleidoscopes, page 402 Chapter 11 Project: Making a Mural, page 588 Chapter 12 Project: Light for All Seasons, page 651

Statistics and Probability

Standard	*Discovering Geometry* Lessons
Interpreting Categorical and Quantitative Data	
Summarize, represent, and interpret data on a single count or measurement variable	
S.ID. 1 Represent data with plots on the real number line (dot plots, histograms, and box plots).	
S.ID. 2 Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.	
S.ID.3 Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).	
S.ID.4 Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets and tables to estimate areas under the normal curve.	
Summarize, represent, and interpret data on two categorical and quantitative variables	
S.ID.5 Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal and conditional relative frequencies). Recognize possible associations and trends in the data.	
S.ID.6 Represent data on two quantitative variables on a scatter plot and describe how the variables are related.	
S.ID.6a. Fit a function to data; use functions fitted to the data to solve problems in the context of the data.	
S.ID.6b Informally assess the fit of a model function by plotting and analyzing residuals.	
S.ID.6c Fit a linear function for scatter plots that suggest a linear association.	(Partial) Chapter 2 Project: Best-Fit Lines, page 107
Interpret linear models	
S.ID.7 Interpret the slope (rate of change) and the intercept (constant term) of a linear fit in the context of the data.	
S.ID.8 Compute (using technology) and interpret the correlation coefficient of a linear fit.	
S.ID.9 Distinguish between correlation and causation.	
Making Inferences and Justifying Conclusions	
Understand and evaluate random processes underlying statistical experiments	
S.IC.1 Understand statistics as a process for making inferences about population parameters based on a random sample from that population.	
S.IC.2 Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation.	

Statistics and Probability	
Standard	***Discovering Geometry* Lessons**
Make inferences and justify conclusions from sample surveys, experiments, and observational studies	
S.IC.3 Recognize the purposes of and differences among sample surveys, experiments and observational studies; explain how randomization relates to each.	
S.IC.4 Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.	
S.IC.5 Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.	
S.IC.6 Evaluate reports based on data.	
Conditional Probability and the Rules of Probability	
Understand independence and conditional probability and use them to interpret data	
S.CP.1 Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or,""and,""not").	
S.CP.2 Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.	
S.CP.3 Understand the conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.	
S.CP.4 Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.	
S.CP.5 Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.	
Use the rules of probability to compute probabilities of compound events in a uniform probability model	
S.CP.6 Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A and interpret the answer in terms of the model.	
S.CP.7 Apply the Addition Rule, P(A or B) = P(A) + P(B) − P(A and B), and interpret the answer in terms of the model.	
S.CP.8 (+) Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B|A) = P(B)P(A|B), and interpret the answer in terms of the model.	
S.CP.9 (+) Use permutations and combinations to compute probabilities of compound events and solve problems.	
Using Probability to Make Decisions	
Calculate expected values and use them to solve problems	
S.MD.1 (+) Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.	
S.MD.2 (+) Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.	
S.MD.3 (+) Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value.	

Statistics and Probability	
Standard	***Discovering Geometry* Lessons**
S.MD.4 (+) Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value.	
Use probability to evaluate outcomes of decisions	
S.MD.5 (+) Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.	
S.MD.5a (+) Find the expected payoff for a game of chance.	
S.MD.5b (+) Evaluate and compare strategies on the basis of expected values.	
S.MD.6 (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).	
S.MD.7 (+) Analyze decisions and strategies using probability concepts (e.g. product testing, medical testing, pulling a hockey goalie at the end of a game).	