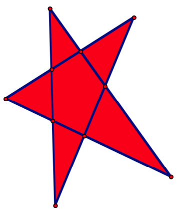
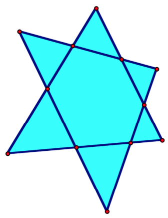
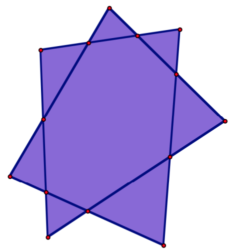
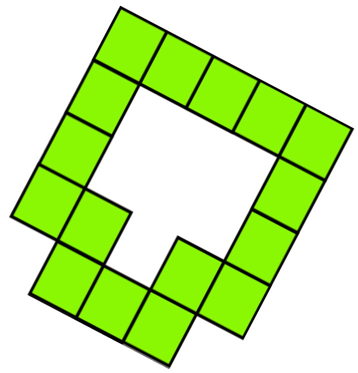
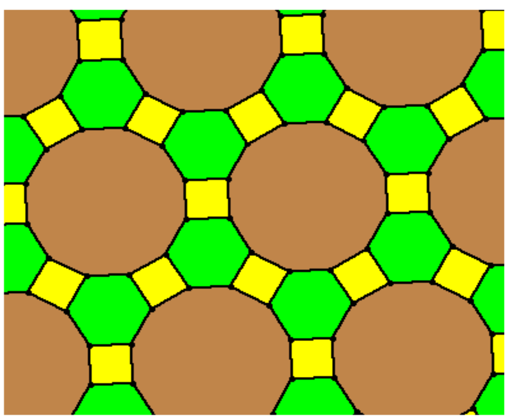
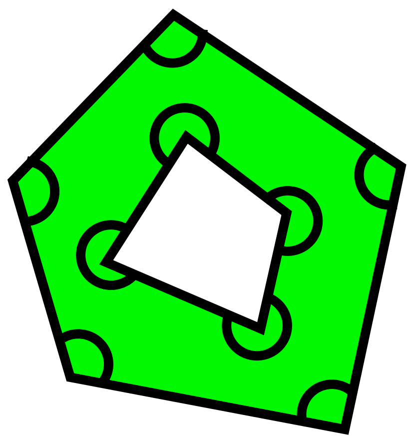
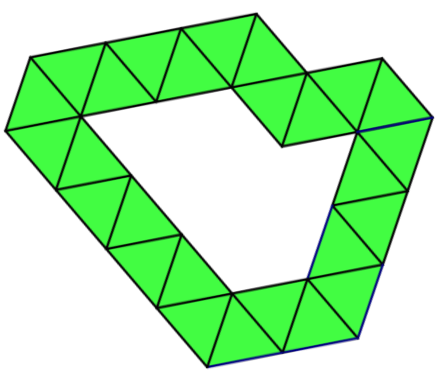
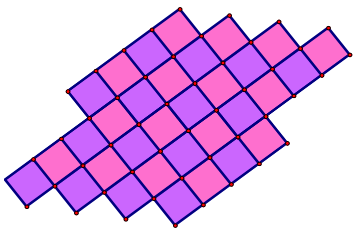
Investigations in Geometry

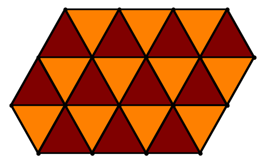








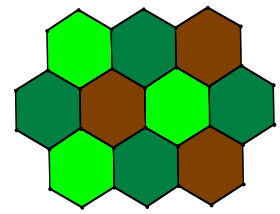
HCTM Maui
October 24, 2015
***with* Michael Serra**

Archimedean Tilings

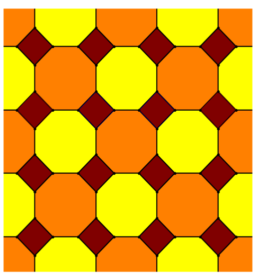
When the same combination of regular polygons meet in the same order at each vertex of a tessellation, it is called an Archimedean tiling or a 1-uniform tiling. There are eleven 1-uniform tilings. Three of the Archimedean tilings are the pure tilings.

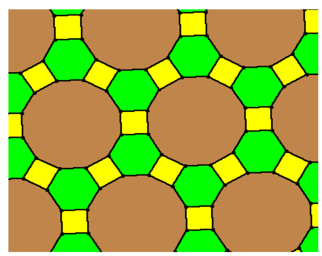


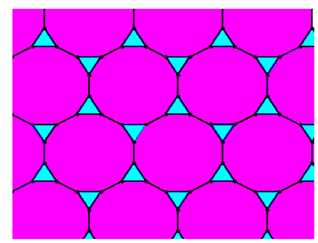




We name 1-uniform tilings by the number of sides of each of the regular polygons that meet at each vertex. What are the names of these 1-uniform tiling?







The remaining five Archimedean tilings use only squares, equilateral triangles, and regular hexagons. Your task is to find the remaining five Archimedean tilings.

The DESE Investigation

Part 1: The investigation

Step 1 On a piece of patty paper, use a double-edged straightedge to draw two pairs of parallel lines that intersect each other.

Step 2 Assuming that the two edges of your straightedge are parallel, you have drawn a parallelogram. Place a second patty paper over the first and copy one of the sides of the parallelogram.

Step 3 Compare the length of the side on the second patty paper with the lengths of the other three sides of the parallelogram. How do they compare? Share your results with your group. Copy and complete the conjecture.

If two parallel lines are intercepted by a second pair, the same distance apart as the first pair, then the parallelogram formed is a ________________.

Part 2: Prove your conjecture

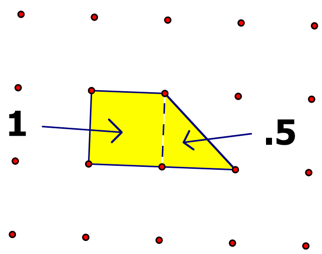
Part 3: Use your conjecture to perform the following DESE constructions:

A. construct the bisector a given angle and explain why it works

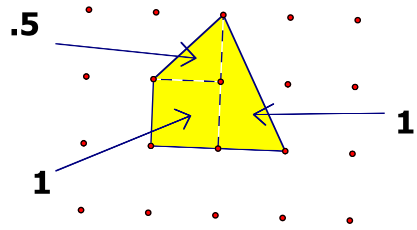
B. construct the perpendicular bisector a given segment and explain why it works

Pick’s formula

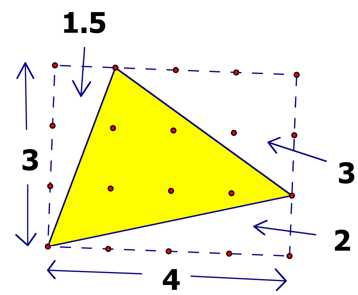
Austrian mathematician Georg Pick discovered a relationship for the area of figures on a square dot grid. The relationship known as Pick’s formula relates the number of interior lattice points and boundary lattice points with the area of the figure. Use the square dot grid on the next page to investigate.



A = 1 + .5
= 1.5



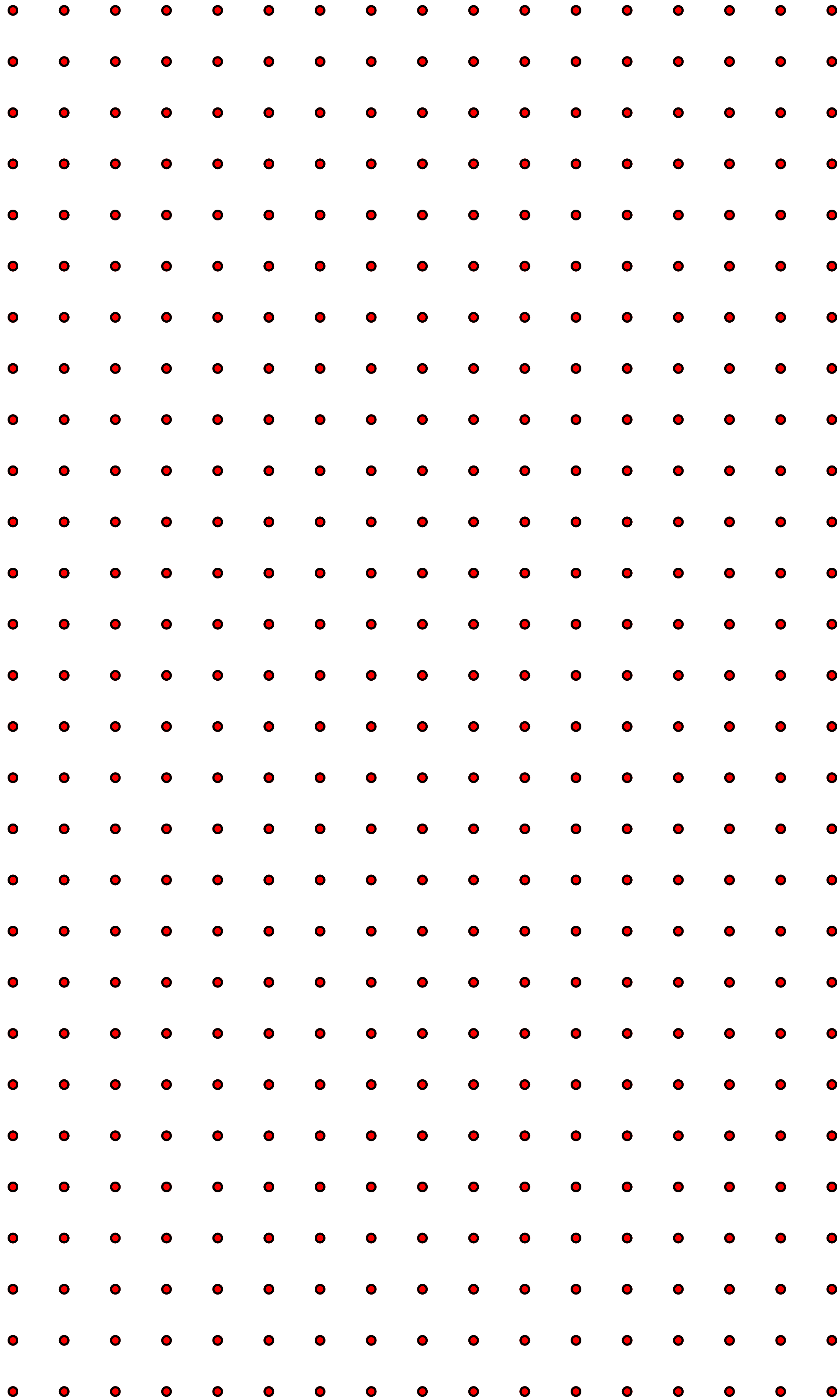
A = 1+.5+ 1
= 2.5



A = 12 – (1.5+2+3)
= 12 – 6.5
= 5.5

# of interior points (i)	# of boundary points (b)						
		3	4	5	6	7	8
	0			1.5			
	1			2.5			
	2						
	3						
	4						
	5	5.5					
		Area of the polygon					

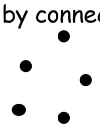
$$A(b,i) = rb + si + t$$

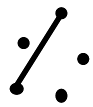


Star Polygons

Star Polygons

Creating a 5-point star polygon by connecting every second point

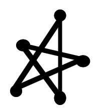




Step 1. Make 5 points

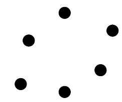
Step 2. Connect 2 points with a segment skipping a point

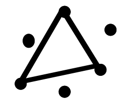
Step 3. Continue connecting every other point.



Special Case Star Polygons

Creating a 6-point star polygon by connecting every fourth point

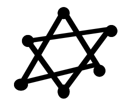




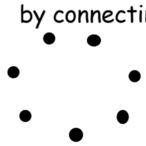
Step 1. Make 6 points

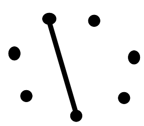
Step 2. Connect every fourth point.

Step 3. Pick another point and repeat.



Creating a 7-point star polygon by connecting every third point

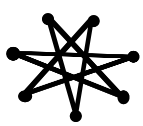




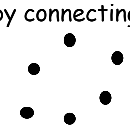
Step 1. Make 7 points

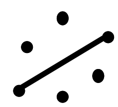
Step 2. Connect 2 points with a segment skipping two points

Step 3. Continue connecting every third point.



Creating a 6-point star polygon by connecting every third point

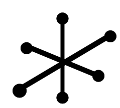




Step 1. Make 6 points

Step 2. Connect every third point.

Step 3. Pick another point and repeat.



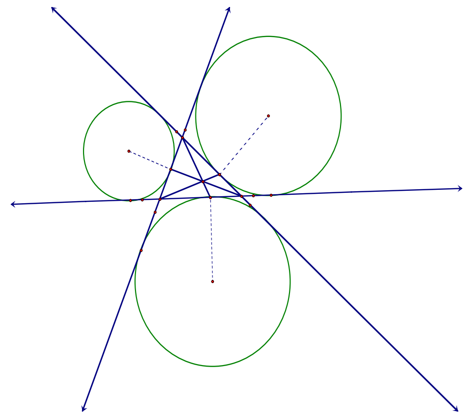
Number of points	Connecting every point	Connecting every second point	Connecting every third point	Connecting every fourth point	Connecting every fifth point
3	180°	180°	-------	-------	-------
4	360°	0	360°	-------	-------
5	540°	180°	180°		-------
6	720°		0		
7	(5)180°				
8	(6)180°				
9	(7)180°				
10	(8)180°				
11	(9)180°				
12	(10)180°				
n	**$(n-2)180°$**				

The Nagel Point

The segments connecting the points of tangency of the three excircles of a triangle with the opposite vertices are concurrent. The point of concurrency is called the **Nagel point**.

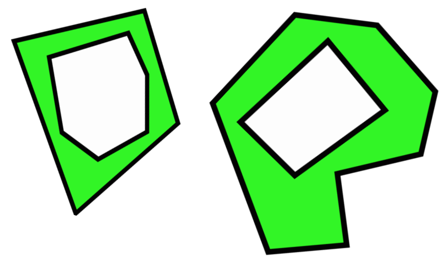
The Nagel point is collinear with two of the other traditional points of concurrency. The segment containing the three is called the ***Nagel Segment***.

What are the three points? How are they positioned on the Nagel Segment?



Donut Polygons

A ***donut polygon*** is the union of two polygons (concave or convex), one in the interior of the other. If the outer polygon has *n* sides and the polygon in its interior has *m* sides then the donut polygon is symbolized by ${}_nP_m$.

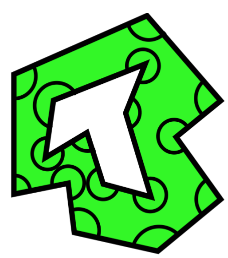


Investigation 1.

What is the sum of the measures of the interior angles of a ${}_nP_m$ donut polygon? By interior angle of a donut polygon we mean an angle measured in the region between the two polygons. Second, what is the relationship between ${}_nP_m$ and ${}_mP_n$?

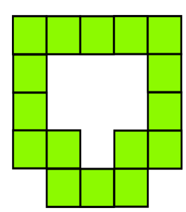
Investigation 2.

What if the concave polygons have two "dents"? What is the sum of the measures of the interior angles of a ${}_nP_m$ donut polygon where at least one or both polygons have two dents?



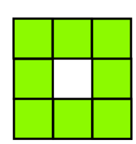
Loop Polygons

A ***Square Loop polygon*** is the arrangement of congruent squares placed side against side so that each square touches exactly two other squares in a complete loop creating an exterior perimeter and an interior perimeter. What is the interior perimeter and the exterior perimeter for a loop of *n* squares?

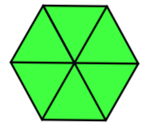


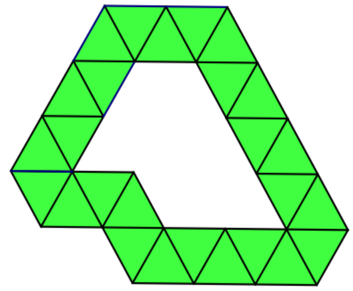
Loop Patterns with Squares

Number in loop	8			16								*n*	100
Inside Perimeter	4			12									
Outside Perimeter	12			20									



An ***Equilateral Triangle Loop polygon*** is the arrangement of congruent equilateral triangles placed side against side so that each triangle touches exactly two other triangles in a complete loop creating an exterior perimeter and an interior perimeter. What is the interior perimeter and the exterior perimeter for a loop of *n* triangles?

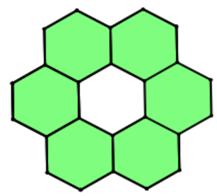


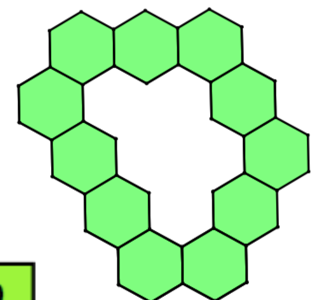


Loop Patterns with Equilateral Triangles

Number in loop	6	26										*n*	100
Inside Perimeter	0	10											
Outside Perimeter	6	16											

A ***Regular Hexagon Loop polygon*** is the arrangement of congruent regular hexagons placed side against side so that each hexagon touches exactly two other hexagons in a complete loop creating an exterior perimeter and an interior perimeter. What is the interior perimeter and the exterior perimeter for a loop of *n* hexagons?



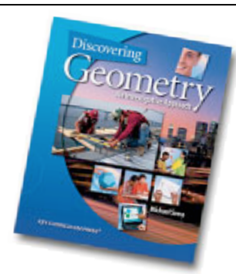


Loop Patterns with Regular Hexagons

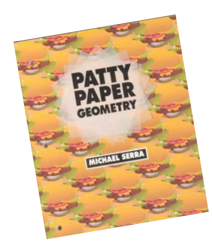
Number in loop	6				11						*n*	100
Inside Perimeter	6				16							
Outside Perimeter	12				28							

Resources

- *Discovering Geometry* 4th edition, Serra, Kendall Hunt Publishing



- *Patty Paper Geometry*, Serra, Playing It Smart 1994



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